GRADE 3 • MODULE 7

Geometry and Measurement Word Problems

New or Recently Introduced Terms

- Attribute (any characteristic of a shape, including properties and other defining characteristics, e.g., straight sides, and non-defining characteristics, e.g., blue)
- Diagonal (e.g., the line drawn between opposite corners of a quadrilateral)
- Perimeter (boundary or length of the boundary of a two-dimensional shape)
- Property (e.g., having all sides equal in length)
- Regular polygon (polygon whose side lengths and interior angles are all equal)
- Tetrominoes (four squares arranged to form a shape so that every square shares at least one side with another square)
- Area (the measurement of two-dimensional space in a bounded region)
- Heptagon (flat figure enclosed by seven straight sides and seven angles)
- Hexagon (flat figure enclosed by six straight sides and six angles)
- Octagon (flat figure enclosed by eight straight sides and eight angles)
- Parallel (lines that do not intersect, even when extended in both directions)
- Parallelogram (a quadrilateral with both pairs of opposite sides parallel)
- Pentagon (flat figure enclosed by five straight sides and five angles)
- Polygon (a closed figure with three or more straight sides, e.g., triangle, quadrilateral, pentagon, hexagon)
- Quadrilaterals (a four-sided polygon, e.g., square, rhombus, rectangle, parallelogram, trapezoid)
- Rectangle (flat figure enclosed by four straight sides, having four right angles)
- Rhombus (flat figure enclosed by four straight sides of the same length)
- Right angle (e.g., a square corner)
- Square (rectangle with four sides of the same length)
- Tangram (special set of puzzle pieces with five triangles and two quadrilaterals that compose a square)
- Trapezoid (quadrilateral with at least one pair of parallel sides)
- Triangle (flat figure enclosed by three straight sides and three angles)

Topic A: Solving Word Problems

In Topic A, students use all four operations to solve one- and two-step word problems within various contexts that were studied throughout the year. The problems are challenging and require students to carefully consider solution paths as they "make sense of problems and persevere in solving them".

Guided practice with strategies for problem solving is built into Lessons 1 and 2. These lessons emphasize the use of modeling through the Read-Draw-Write (RDW) process and revisit models such as tape diagrams and number bonds. Students flexibly use a letter to represent the unknown as they solve. This readies them for problem solving with perimeter and area in Topics C and E.

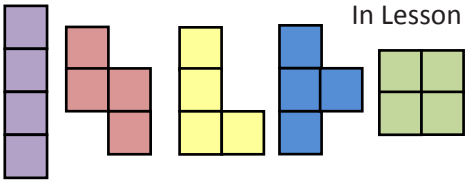
In Lesson 3, students' level of independence within the lesson increases. They work together or on their own to develop solution paths, and then share strategies and solutions. Students think critically about their own solutions and the work of others as they review and critique one another's work. They discuss the clarity, practicality, and efficiency of different models and strategies, refining their own understandings and approaches. Student presentations of work, protocols for critiquing, and gallery walks are structures that provide a platform for this dialogue.

Topic B: Attributes of Two-Dimensional Figures

In Topic B, students use their understanding of geometry from Grade 2 to explore quadrilaterals. In Lesson 4, they learn that different shapes (e.g., squares, rectangles, and rhombuses) have shared attributes that can fall within a larger category (parallelograms, quadrilaterals, and trapezoids). They explore these new, larger categories and understand, for example, that any quadrilateral can be decomposed into two triangles. As they learn which attributes are shared, the process of comparing shapes also leads to discussion about the differences between shapes; students learn, for example, that not all rectangles and rhombuses are squares.

Students use their understandings of the attributes of quadrilaterals to compare other polygons in Lesson 5. They look for shared attributes and learn to recognize polygons with sides that are equal—regular polygons—which helps lay a foundation for problem solving with perimeter in later topics.

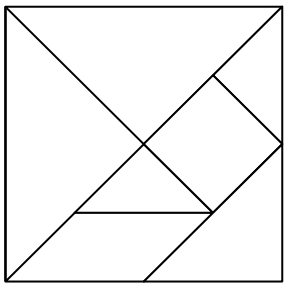
While students analyze the attributes of given shapes in Lessons 4 and 5, in Lesson 6 they draw shapes based on given attributes. For example, students may be asked to draw a quadrilateral with at least two right angles and talk about which shapes are possibilities. They also draw quadrilaterals that do not fit any subcategories. Prompts such as "draw a polygon with only two sides and two angles" spark investigative discussion through which students determine the impossibility of such a shape. This lesson helps students solidify their intuitive understanding of polygons.



Tetrominoes

In Lesson 7, students work with tetrominoes. They use grid paper to construct a set, then reason about how to create larger shapes, such as rectangles, using them. This develops spatial structuring skills by way of manipulating and composing shapes.

Students use their experience with composing shapes to help them *decompose* a square to create a tangram puzzle (pictured to the right). Lesson 8 guides students through the process of decomposing, and then reconstructing, the original square using the seven puzzle pieces.



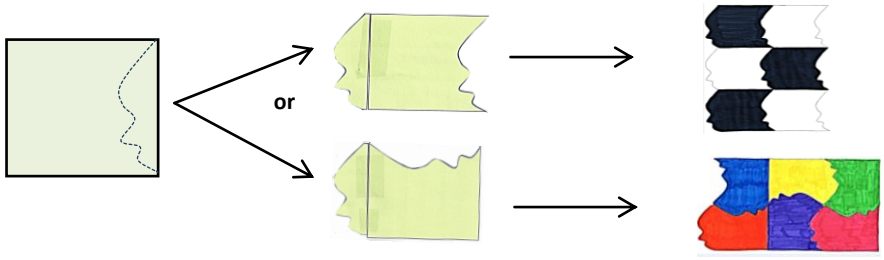
Tangram

In Lesson 9, students learn to analyze relationships between tangram pieces. For example, students might discover that the two largest triangles compose one larger triangle, or that the two smallest triangles can be manipulated to compose the small square, parallelogram, and medium triangle.

Students solve tangram puzzles using their pieces and discuss whether or not there is more than one way to compose a given polygon. Describing their strategies provides engaging context for using the vocabulary of attributes: "I found that the right angle of the small triangle forms the top of the duck's head." Students may create their own interesting polygons and trade with partners to see if a peer can use their tangram pieces to complete the outline.

Topic C: Problem Solving with Perimeter

Students are introduced to perimeter in Topic C. Conceptual exploration begins by creating tessellations. In Lessons 10 and 11, students decompose a quadrilateral. They rearrange the parts to form a new shape. They then use the new shape to tile, tracing its perimeter until a new larger shape (the complete tessellation) is formed. Through this work, students define perimeter as the boundary of a two-dimensional shape and use their new vocabulary in context as they describe the process of tessellating. This lesson begins the study of perimeter with unusual shapes to encourage flexible thinking about perimeter and avoid the misconception that it is a property of rectangles alone.



Cut on the line. Then, slide the piece to the opposite side or rotate it to an adjacent side to make a new shape.

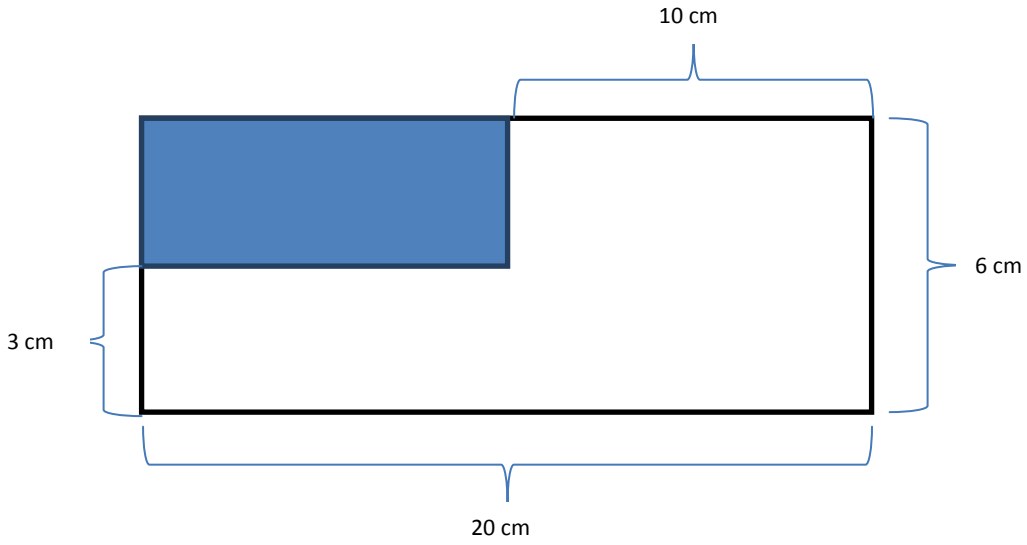
In Lesson 12, students measure side lengths and calculate perimeters. They measure the side lengths of polygons (in whole number units) using rulers and then use these side lengths to determine perimeter. Students attend to units as they solve and discuss the efficiency of strategies for adding side lengths. The next complexity is that students are given pictorial models, including the side lengths of polygons, from which they determine the perimeter in Lesson 13.

Lesson 14 provides more complex problem solving; students determine the perimeter of a figure when whole number side length measurements are missing. Students use their knowledge of attributes of shapes to fill in missing information, and then calculate the perimeter. For example, they may be told that a hexagon is regular and that one side length is 5 centimeters. Based on that information, students fill in missing side lengths and calculate the perimeter, discussing whether addition or multiplication is a more efficient strategy for solving the problem.

In Lesson 15, students apply their basic understanding of perimeter to real world contexts. They explore how perimeter is used in everyday life, and they develop strategies for calculating perimeters using known information.

Lesson 16 extends students' knowledge of perimeter to circles. In this lesson, students rotate through stations and wrap string around various circular objects, such as lids. Students measure their strings to the nearest quarter inch using rulers and record their measurements for comparison and discussion. This lesson reinforces that perimeter is a measureable attribute for any shape, not just polygons, and that those measurements can occur in both whole and fractional units.

Lesson 17 involves using all four operations to determine a perimeter and any missing measurements. Students develop strategies for finding part of a larger shape, for example, the blue rectangle in the figure below. In this example, students understand that they can subtract the known part of the length from the total length to find the missing measurement. The missing measurements may then be used to find the perimeter of the blue rectangle.

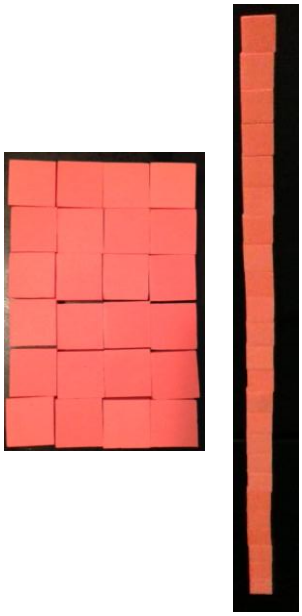


What is the perimeter of the blue rectangle?

Topic D: Recording Perimeter and Area Data on Line Plots

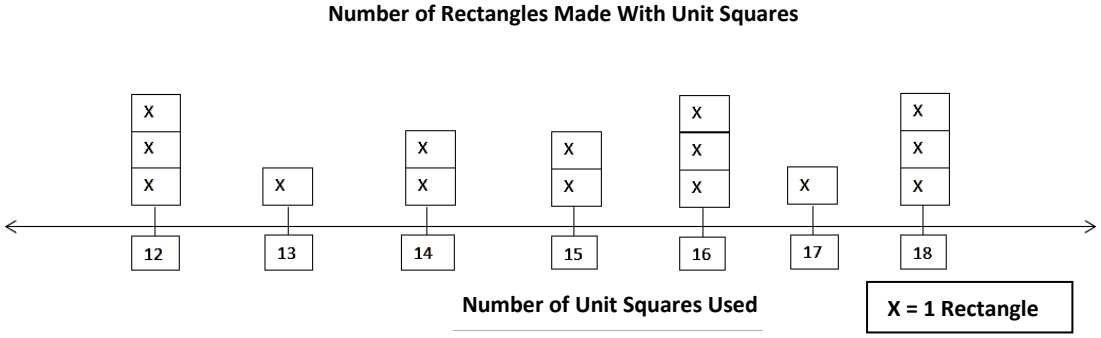
In Topic D, students utilize a line plot to draw conclusions about perimeter and area measurements.

Students use a given number of unit squares to build and determine different perimeters of rectangles in Lesson 18. For example, given a rectangle composed of 24 unit squares, students find there are four possible perimeters: 50, 28, 22, and 20 length units. They draw their rectangles on grid paper and discuss the fact that rectangles with side lengths that are equal or almost equal (squares or square-like rectangles) have smaller perimeters than rectangles whose side lengths are very different (long and narrow rectangles). Students continue to explore with different numbers of unit squares and record the number of possibilities, noting when they have found all the possible combinations. They recognize that area and perimeter are measured in different units and conclude that, in general, there is no way of knowing an exact perimeter for any number of unit squares without more information about the side lengths.

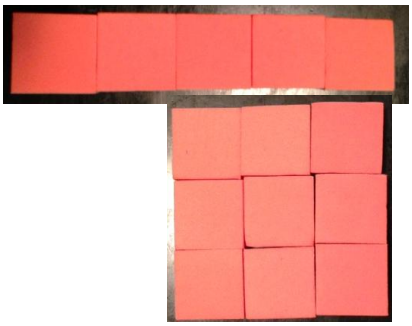


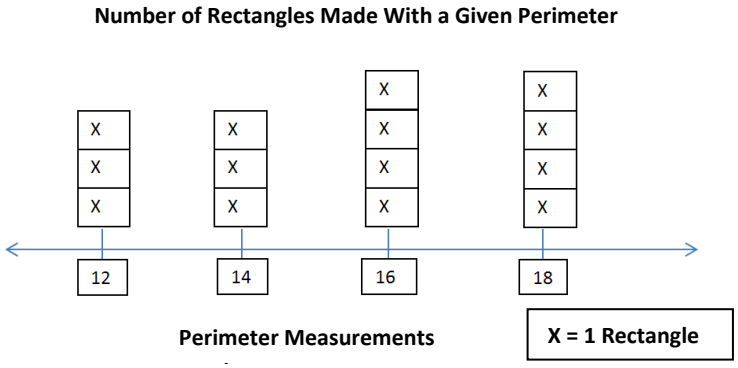
In Lesson 19, students use a given number of unit squares to make all possible rectangles. They construct line plots showing the number of rectangles they constructed for each number of unit squares. Students analyze the line plot and draw conclusions based on the data. They discuss why some numbers of unit squares, such as 13,

produce only one possible perimeter.



Using understanding that perimeter is double the sum of the length and width, in Lessons 20 and 21, students find the different areas of rectangles made with unit squares and a given perimeter. For example, they are asked to build rectangles with a perimeter of 12 unit squares and divide 12 by 2 to find that the sum of the length and width is 6. Students then determine that they can make three rectangles whose lengths and widths add to 6, which results in rectangles made with 5, 8, or 9 unit squares. Students discuss differences in the areas of rectangles with the same perimeter. They record their findings for use in Lesson 22, when they again construct a line plot and draw conclusions about the data.





Topic E: Problem Solving with Perimeter and Area

In Topic E, students solve problems with perimeter and area. Lesson 23 focuses on solving a variety of word problems involving perimeter. This provides students an opportunity to use multiplication and division strategies to solve problems about perimeter.

Students use rectangles and circles to measure, create, and analyze robots and environments for the robots using specified perimeter measurements in Lessons 24 through 27. They reason about the different whole number side lengths that may be produced. For example, when given the requirement that the perimeter of the arms of the robot must be 14 inches, students experiment and draw different possibilities for rectangles to determine which ones they prefer for the robot's arms based on their explorations with unit squares in Topic D. Students cut out and assemble the parts of the robot from grid or construction paper, and compare their robots with their peers'. The final lesson in this sequence provides an opportunity for peer review and critique, and also for discussion about the difference between the areas of robots and their environments despite the fact that they have the same given perimeters.

Students return to problem solving in Lessons 28 and 29, this time working with a variety of word problems involving both area and perimeter. For example, if students are given both the length and the width of a rectangular football field, they should be able to determine both the perimeter and the area of the field. In these lessons students explore and develop strategies for solving a sequence of increasingly complex problems. In Lesson 30, students further develop analyzing and critiquing skills. They initially discuss anonymous student work samples before sharing their own work and receiving feedback in small groups.